MULTI-DIMENSIONAL ANALYSIS OF TMD LETHALITY DATA

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Abstract

PEELS (Parametric Endo- Exo-atmospheric Lethality Simulation) calculations involve lethality determinations that typically include more than two parameters. A method of interpreting such multi-dimensional lethality results is presented using multiple regression techniques. A PEELS body-to-body calculation was run where the relative velocity, the strike angle, the impact point, and the overall density and length to diameter ratio of a cylindrical kill vehicle were varied. The mass of the kill vehicle was set at 15 kilograms and the threat was the unclassified PEELS TBMCS5 (STORM) target vehicle with a payload of 38 chemical submunitions. The parameters were considered to predict the lethality from hits, $P_{k/h}$, as polynomial factors in a regression equation. The resulting multiplication results in a very large number of terms that contain, as products, every combination of the variables in the factors. A number of reduced regression equations were set up that included the original parameters as well as a few of the cross terms. These equations were then subjected to multiple linear regression to examine the significance of the variable terms. Additional regression equations were set up to examine the specific functional relationship of one or two parameters (linear and quadratic) to the lethality. One of the unexpected results was, for this configuration, that the lethality was not a strong function of the relative velocity and the most effective kill vehicle had the lowest density and hardness.

Introduction

The availability of a high accuracy, fast running lethality code, such as PEELS, has made the determination of kill probability, of a kill vehicle against a TBM (tactical ballistic missile), easily done, even for a complicated set of engagement parameters. PEELS is so fast, it is not unusual for a single problem to have hundreds of thousands of sample runs. The possibility of generating such huge amounts of data presents the opportunity of very thorough analyses of TMD engagements over a wide range of engagement parameters. The down side of this opportunity is that the analysis of such data can be very time consuming, due to the problems of cognitively encompassing such a wealth of data. The usual way of perceiving the interrelationship between two independent engagement parameters and a measure of lethality is a 3-D surface plot or the equivalent 2-D contour plots. But the consideration of more than two independent parameters varying at a time is cumbersome or even impossible to visualize. Therefore an efficient method of analyzing such data in a manner that makes the important relationships clearer is desirable.

One way to do this is using multiple regression techniques. Multiple regression will handle massive amounts of data with a large number of parameters. Using multiple regression the effects of the multiple independent variable parameters on lethality can be quantified in a manner that permits conclusions to be drawn about the individual effects of each parameter, as well as the interactive effects of the parameters on lethality. Such an analysis, besides being helpful in the usual lethality trade studies, can be utilized to aid the design of hardware used in the endgame, such as sensor algorithm optimization, and kill vehicle sizing. This paper shows the use of regression techniques, using data generated by PEELS, for the interception of a generic TBM by a simplified kill vehicle.

TMD lethality can be defined as the probability of negating a threat missile given a hit by a TMD kill vehicle. In the context of this paper, the threat missile is a generic TMD threat containing submunitions, and the probability of negation is then defined as the fractional kill of the submunitions (FKS), as a decimal. Kill itself is defined within the operation of the algorithmic computer code, PEELS. Although it is not intended to discuss the particular way that PEELS predicts kill, the lethality is derived from the kinetic energy effects of body-to-body impact of the kill vehicle on the threat, at the conclusion of the end game between the two. The end game is described by a number of parameters that relate to the encounter variables (relative velocity, strike angle, kill vehicle and threat pitch, yaw, and roll, kill vehicle and threat reference point, altitude, impact point, and warhead, or

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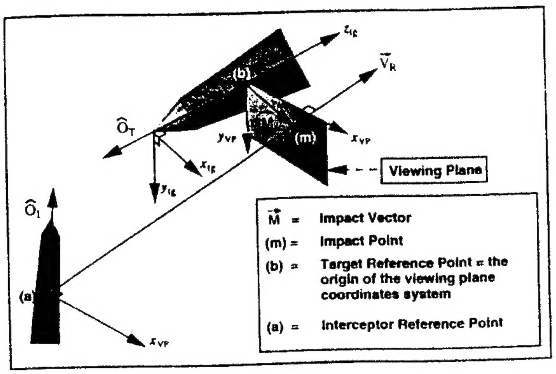
kill enhancement device (KED), properties, if any). This paper will not consider either warheads or KED's, and further will limit the analysis to exo-atmospheric endgames where the threat axis is along its velocity vector and the kill vehicle axis is along the relative velocity vector. This is not a limitation due to PEELS or multiple regression, but only to keep the present analysis relatively simple and the relationships developed fairly uncomplicated. Therefore the encounter variables that will be included in the analysis are the relative velocity, the strike angle, and the impact point. Figure 1 is taken from the PEELS user's manual[1]. It shows the relationship between these encounter variables and the kill vehicle and threat reference points.

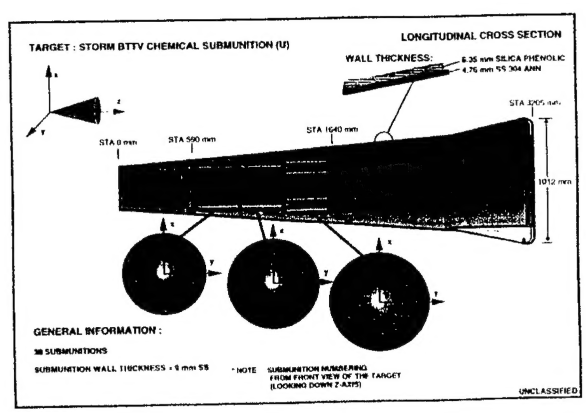
Fig.1 PEELS Viewing Plane and Coordinate System



The position of the reference points is arbitrary and can be placed anywhere on the axes of the two bodies; often the default positions are at the center of mass of the kill vehicle, and somewhere in the payload area of the threat. The strike angle is defined as the angle between the relative velocity vector, V_R, and the threat velocity vector. The relative velocity vector is the kill vehicle velocity vector minus the threat velocity vector. There is a plane, called the viewing plane, that is perpendicular to the relative velocity vector and passes through the threat reference point. The threat reference point is the origin (X,Y) of the viewing plane coordinate system. The impact point is defined as the intersection of the relative velocity vector and the viewing plane. Assuming no deflection of the kill vehicle during the interaction with the threat, the reference point of the kill vehicle will pass through the impact point. The projection of the threat on the viewing plane is what an observer, at the kill vehicle reference point, will see if he looks along the relative velocity vector. The threat will appear as a nose view, a side view, or an end view as the strike angle varies from 0 to 90 to 180 degrees. Although the impact point may be anywhere in the view plane, for this analysis the impact point will lie on the projection of the threat axis on the viewing plane. This means that the impact point needs only one quantity, say X, to define it. There will often be off axis ($|Y|>0$) impact points in real encounters, but generally, the peak of the lethality contours will be along the axis. Thus, in choosing the most lethal impact point, Y=0 is usually a good choice, whereas, the best choice of X is more problematical. Since the length of the threat axis in the view plane will vary with strike angle, the definition of the impact point, in length units, while accurate, is difficult to visualize. Thus, the impact point variable (XR) was defined as the distance, along the projected axis of the impact point from the tip of the threat (or the forward projected edge) divided by the projected length of the threat. Note that the projected threat length is not the same as the projected axis length, except for a 90 deg strike angle. For a 0 or 180 deg strike angle, the projected threat length is equal to the maximum threat diameter, and the forward projected edge is at one end of the diameter. However XR is the same for all projections.

The other factors related to the lethality of an encounter are the threat and the kill vehicle themselves. This analysis will use as the threat, the unclassified PEELS generic threat denoted as TBMCS5, Figure 2.

Fig. 2 PEELS Target TBMCS5



This threat has 38 submunitions as a payload located almost entirely in the front half of the threat. The threat is 320.5 cm long, and is a little over 101 cm in diameter, at the rear. The front of the threat is a truncation, about 34 cm in diameter. The analysis will consider the effect of nine different kill vehicles, all solid uniform cylinders, and all of the same mass. The variables defining the cylinders are the density of the

cylinders and their length to diameter ratios (L/D). The set of variables for the analysis therefore is :

Table 1. Basic variables

Variable	Symbol
Fractional kill of the submunitions	FKS
Relative velocity (km/sec)	V
Strike angle (deg)	SA
Impact point projection ratio	XR
Kill vehicle density (gm/cc)	Rho
Kill vehicle length to diameter ratio, L/D	LoD

Kill Vehicle

The mass of the kill vehicle was set at 15 kgms. The densities were set at 0.5, 1.0, and 1.5 gms/cc. The L/D ratios chosen were 0.488, 1.382, and 3.908. These are all fairly reasonable, but arbitrary values. The kill vehicles were all cylindrical in shape. The PEELS kill vehicle input file was generated using the PEELS CMODEL program. The Brinell hardness (H) of the kill vehicle material was estimated by:

$$H = 100*(Rho/2.7) \qquad (1)$$

The set of kill vehicles appears in the table, below.

Table 2. KV Properties

Designation	Rho(gms/cc)	L/D
KV1	0.5	0.488
KV2	0.5	1.382
KV3	0.5	3.908
KV4	1.0	0.488
KV5	1.0	1.382
KV6	1.0	3.908
KV7	1.5	0.488
KV8	1.5	1.382
KV9	1.5	3.908

Encounter Set

Since a multiple regression analysis minimizes the squared differences between the predicted and actual dependant variable, the analysis is most useful when it uses reasonable values of the independent variables. There is not much use to obtaining a good fit over unlikely portions of the independent parameter space. The best set of the encounter variables, velocity and strike angle, would be the set of values that would be most likely to occur. This could be obtained from an accurate fly out analysis. The analysis would generate velocity and strike angle pairs and these would be used in the multiple regression. Such a set was not available, so instead, an analytical encounter set was used. Since this paper is focused on illustrating methods rather than producing real world results, such an artificial parameter set is appropriate. The encounter set, or simulated intercept space, used was taken from the paper by J. L. Lamb[2]. The encounter set is obtained by applying Weibull probability distributions along perpendicular directions (u-v) defining a rectangular threat engagement zone in the strike angle-relative velocity plane. The Weibull distributions are the same as in Lamb's paper. The origin of the u-v coordinate system is at V=1.75 and SA=144.0. Thus the velocity will never be less than 1.75 km/sec and the strike angle is very unlikely to be larger than 158 degs. The transformation from the u-v coordinate system to the V-SA coordinate system is given by Lamb as

$$\begin{Bmatrix} V \\ SA \end{Bmatrix} = \begin{bmatrix} 5 & 0 \\ 0 & 180 \end{bmatrix} \left[\begin{Bmatrix} 0.35 \\ 0.80 \end{Bmatrix} + \begin{bmatrix} \cos(\alpha) & -\sin(\alpha) \\ \sin(\alpha) & \cos(\alpha) \end{bmatrix} \begin{Bmatrix} u \\ v \end{Bmatrix} \right] \qquad (2)$$

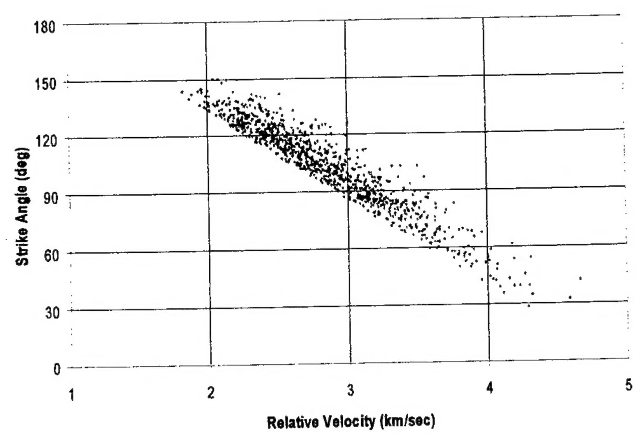
where α = -0.915 radians.

A 1000 pair sample was generated using the above distribution. The pairs are plotted in Fig. 3. The regression line found for the points was

$$SA = 229.8 - 44.1\ V \qquad (3)$$

The correlation between SA and V was –0.943. The mean and standard deviation for V and SA were 2.83 (km/sec) and 0.46 (km/sec), and 105.1 deg and 21.5 deg, respectively.

Fig. 3 Simulated Intercept Space



Impact Points

The impact point ratios (XR) were generated by means of a uniform random generator giving values between 0 and 1.0. PEELS requires an actual distance as a input variable so to obtain this, the threat reference point was set at the tip, and then the impact point as the distance from the tip is obtained by multiplying XR by the length of the threat in the view plane. The hit point on the threat is obtained by multiplying XR by the length of the threat.

Regression Equation

The regression equation was obtained by assuming that the appropriate equation would be composed of factors. Each factor would be a function of only one independent variable. To keep the equation manageable, initially, only linear and quadratic factors were used. Quadratic factors were used where there was some chance that the variable would produce a local maximum or minimum, and linear factors, where such was not likely. Only relative velocity was deemed not likely to have either a local minimum or maximum so it was a linear function and the rest were quadratic functions. Thus the regression equation was

$$FKS = F_1(V)*F_2(SA)*F_3(XR)*F_4(Rho)*F_5(LoD)\,, \quad (4)$$

Where,

$$F_1(V) = c_1+c_2*V \quad (5)$$
$$F_2(SA) = c_3+c_4*SA+c_5*SA^2 \quad (6)$$
$$F_3(XR) = c_6+c_7*XR+c_8*XR^2 \quad (7)$$
$$F_4(Rho) = c_9+c_{10}*Rho+c_{11}*Rho^2 \quad (8)$$
$$F_5(LoD) = c_{12}+c_{13}*LoD+c_{14}*LoD^2\,, \quad (9)$$

And all the c's are constants.

If all the factors are multiplied out, with the nine variable terms there are hundreds of combinations of products of the linear and quadratic terms. Many of these do not significantly add to the determination of FKS. The basic regression equation is represented by

$$FKS=k_1+k_2*v+k_3*SA+k_4*XR+k_5*Rho+k_6*LoD+k_7*SA^2+k_8*XR^2+k_9*Rho^2+k_{10}*LoD^2+\ldots, \quad (10)$$

where the product terms follow, and the k's are constants.

PEELS Analysis

PEELS was run in the PK option mode for body-to-body impact[1]. The input data file structure consists of a row of the following variables: Ncase, Nsample, the speed of the kill vehicle (which is set negative to indicate to PEELS that the engagement is body-to-body impact), the x,y,z components of the kill vehicle velocity unit vector (in threat coordinates), the x,y,z components of the kill vehicle reference point location with respect to the threat reference point (in threat coordinates), the x,y,z components of the kill vehicle orientation unit vector (in threat coordinates), and the altitude.

The threat reference point was located at the tip (the origin of the threat coordinate system). The speed was set equal to –V. The x,y,z components of the kill vehicle velocity unit vector were computed using SA and the assumption that the vector was in the x-z plane. The x,y,z components of the kill vehicle reference point location were calculated by confining the location to the x-axis (except for the case SA=90 deg) and calculating the x and z coordinates using SA and the hit point found from XR. In the case where SA = 90 deg, the kill vehicle reference point location was put along a line parallel to the x-axis, passing through the hit point. Since the kill vehicle is oriented along the relative velocity vector, the x,y,z components of the kill vehicle orientation unit vector are the same as the x,y,z components of the kill vehicle velocity unit vector. The altitude is conveniently set at zero, since it doesn't effect these calculations.

There were nine combinations of the fixed values of Rho and LoD, corresponding to the nine kill vehicles. For each kill vehicle, 1000 sets of PEELS input variables were generated. For each set, a V-SA pair was generated using the u and v Weibull distributions. First a uniform random variable, w, was generated between 0 and 1.0. then a random Weibull variable, u (or v) is calculated from

$$u = r * \exp\left[\frac{1}{m}\ln\left[-\ln(1-w)\right]\right] \quad (11)$$

where r and m are parameters in the Weibull distribution

$$p(u,m,r) = \frac{m}{r}\left(\frac{u}{r}\right)^{m-1} e^{-(u/r)^m} \quad (12)$$

The u and v pair are then used to compute a V and SA from (2). A random impact point ratio, XR, is generated and then V, SA, and XR are used to produce the PEELS PK option input file data rows.

Regression Analysis

The initial analysis considered 10 terms of (10) consisting of the basic linear and quadratic terms. This gave a coefficient of multiple correlation, R, and R^2 of about 0.74 and 0.54, respectively. R is a measure of the correlation between the predicted and measured

(PEELS calculated) FKS values. R^2 is a measure of amount that the variability of FKS is accounted for by the regression equation. These values were lower than desired, so additional terms (the cross terms) were added to the regression equation. However, even increasing the number of terms to 30, gave very little increase in R and R^2 (about 0.76 and 0.57, respectively). In an effort to find additional terms that might give significant increases in R and R^2, cubic terms were added to the regression equation. The cubic terms gave a significant increase to R and R^2 (to about 0.87 and 0.76 respectively). Further analysis showed that almost all of this increase was due to the XR^3 term, and that XR^4 did not significantly improve R. It was also found that the Rho^2 term was not significant. The final regression equation was, then,

$$FKS=k_1+k_2*V+k_3*SA+k_4*XR+k_5*Rho+k_6*LoD+ k_7*SA^2+k_8*XR^2+k_9*LoD^2+k_{10}*XR^3 \quad (13)$$

The betas and B's found for the regression equation are given below.

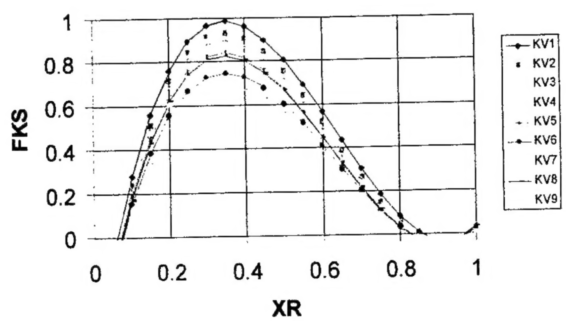
Table 3. Regression Coefficients

Variable	beta	B
V	0.0634	0.0567
SA	-0.8026	-0.0154
XR	6.5923	9.4212
Rho	-0.0630	-0.0638
LoD	-0.2123	-0.0605
SA^2	0.9040	0.000086
XR^2	-13.9846	-18.7527
LoD^2	0.1176	0.0072
XR^3	7.1998	9.9204

The intercept, k_1, equals 0.03306. The B's are the other coefficients (k_2 = 0.0567, etc.). The betas are a measure of the importance of the variable in the regression equation, adjusted so that the variables are on the same scale. It is no surprise that the impact point variables are the most important determinants of FKS. The largest beta is for XR^2, showing a very strong quadratic element to the impact point. The strike angle variables are next in importance, followed by the L/D variable, both with a substantial quadratic element. As can be seen, V and Rho are the least important variables in the equation. They should not be excluded, though, since all the variables are significant at the $p<0.001$ level. This equation estimates FKS for the included variables, but the variables should be kept within the limits that were initially used to calculate FKS in PEELS. Since (13) is an estimate of FKS, the equation may produce values for FKS outside the range of 0 to 1.0, for some combination of the variables.

The regression set was made up of nine sets corresponding to the nine KV's. A regression of each of the nine sets on the XR variables produces the set of nine curves shown in Fig. 4, for FKS vs. XR for each of the nine KVs. Although the cubic term was required to provide a good fit to FKS, the slight upturn of FKS near XR=1.0 is probably an artifact of the functional form, and does not represent a physical effect. Note that KV1, the lowest density, lowest hardness KV shows the highest, widest lethality curve. This, however does not mean that low density correlates directly with high lethality.

Fig. 4 FKS vs. XR for All KVs



If the KVs are sorted according to decreasing diameter, the result appears in Table 4.

Table 4. KVs Ordered According to Diameter

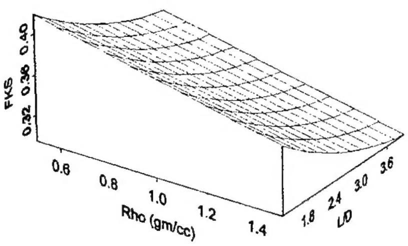
KV#	Diameter (cm)	Rho (gms/cc)	L/D
1	42.76	0.50	0.488
4	33.94	1.00	0.488
2	30.94	0.50	1.382
7	29.65	1.50	0.488
5	24.00	1.00	1.382
3	21.38	0.50	3.908
8	20.97	1.50	1.382
6	16.97	1.00	3.908
9	14.83	1.50	3.908

The peaks in Fig 4 are ordered, according to KV #, as, 1,2,4,7,5,3,8,6,9. So the order is almost according to Table 4, except that in the switch between KVs 4 and 2, there is a slight preference for the lower density. The diameter effects are predominant, but,

unexpectedly, lower densities give a slight increase in lethality.

A regression analysis was made for Rho and LoD only. The results are shown in Fig. 5.
There is a small preference for both low density and low L/D, that is, probably, mostly a diameter effect. The quadratic term for LoD is evident.

Fig. 5 FKS vs. KV Rho and L/D



Summary and Conclusions

The lethality code, PEELS, can generate a wealth of data for the lethality of a KV on a TMD threat. This data, for which there may be a large number of associated variables, can be cumbersome to analyse by ordinary inspection or graphical methods. A PEELS calculation was made to provide FKS as a result of five variables. A regression analysis provided an equation for FKS as a function of the five variables, where all the function relationships were significant at the $p<.001$ level.

The regression analysis also provided the following:

- a means of separating significant from unsignificant variables postulated to be terms in the regression equation for FKS;
- an ordering of the variables with respect to their influence on FKS;
- a means of separating out the effect of a single variable, or set of variables;
- a means of assessing the effects of less significant variables.

This last benefit is more important than it may seem. Generally the sought after lethality level is close to 1.0, but the levels very close to 1.0 are difficult to attain. Even if a variable has a small effect on FKS, if its effect adds on to the top of the lethality level attained, as in Fig. 4, it can be very important.

The expression of lethality as an equation means that those groups assessing or designing different aspects of the interceptor, such as the KV design, the flyout, or the sensor algorithms, can easily see the effects of their work on endgame lethality.

References

1. PEELS Users Manual, Version 7.1, Aug. 1, 1997, Lethality Division of Weapons Directorate, U.S. Army Space and Strategic Defense Command.

2. Lamb, J. L., "A Practical Probabalistic Analysis Method for Hydrocode-Based Lethality Assessment.", presented at the 7th Annual AIAA/BMDO Technology Readiness Conference, 3-7 August 1998.

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